

MODAL AUDIO EFFECTS: A CARILLON CASE STUDY

Elliot Kermit Canfield-Dafilou

Center for Computer Research in Music and Acoustics,
Stanford University, Stanford, CA 94305 USA
kermit@ccrma.stanford.edu

Kurt James Werner

The Sonic Arts Research Centre
Queen’s University, Belfast, Northern Ireland
k.werner@qub.ac.uk

ABSTRACT

Modal representations—decomposing the resonances of objects into their vibrational modes has historically been a powerful tool for studying and synthesizing the sounds of physical objects, but it also provides a flexible framework for abstract sound synthesis. In this paper, we demonstrate a variety of musically relevant ways to modify the model upon resynthesis employing a carillon model as a case study. Using a set of audio recordings of the sixty bells of the Robert and Ann Lurie Carillon recorded at the University of Michigan, we present a modal analysis of these recordings, in which we decompose the sound of each bell into a sum of decaying sinusoids. Each sinusoid is characterized by a modal frequency, exponential decay rate, and initial complex amplitude. This analysis yields insight into the timbre of each individual bell as well as the entire carillon as an ensemble. It also yields a powerful parametric synthesis model for reproducing bell sounds and bell-based audio effects.

1. INTRODUCTION

The modal approach conceives of resonant objects and rooms by their modes of vibration, where each mode is characterized by a frequency, decay rate, and complex amplitude [1]. Historically, this concept has informed acoustic analysis of both interiors [2] and musical instruments [3], and has informed digital audio synthesis techniques including the Functional Transformation Method [4], MODALYS [5], MOSAIC [6], and advanced numerical modeling of strings [7–9] and bridges [10]. Recently, modal analysis of rooms has informed a new family of artificial reverberation algorithms called “modal reverb,” where a room’s modal response is synthesized directly as a sum of parallel filters [11, 12]. Modifications to the basic modal reverb algorithm have been used to produce novel abstract audio effects based on distortion, pitch, and time-scale modification [13] as well as an abstract Hammond-organ-based reverberation algorithm [14].

In this paper, we show that extensions to a modal *synthesis* algorithm based on analysis of recordings can also be used to produce interesting and musically useful abstract audio synthesis algorithms. We use recordings of 60 bells of the Robert and Ann Lurie Carillon at the University of Michigan as a case study of this approach [15]. Because bells are struck by a metal clapper, the driving force is impulsive. It excites a large number of inharmonic modes to resonate and decay exponentially.¹ A modal representation provides an intuitive interpretation for bell-modeling. Moreover, the model is well suited for manipulations allowing us to augment bell-sound synthesis for extended audio processing.

The University of Michigan released the audio recordings of one of their carillons as part of an initiative to promote contem-

porary composers to write electroacoustic music for their carillon. While the samples could be manipulated in a variety of ways, a parametric model of the carillon is useful for producing an extended range of bell-based sounds and effects. Our modal analysis yields insight into the timbre of each individual bells as well as the entire carillon as an ensemble, and provides a powerful parametric synthesis model.

Carillons date back to the middle of the seventeenth century. A carillon is a musical instrument consisting of at least 23 bells that are tuned chromatically and played with a baton keyboard interface [16].² Carillons are usually held in towers and are the second largest instrument following the pipe organ. Over the last 350 years, bell casting technology has improved and musical sensibilities have changed. The Lurie Carillon consists of sixty Royal Eijsbouts bells that were cast in 1995–6 in the Netherlands [17, 18]. It is one of the largest, heaviest, and youngest carillons in the United States.

Carillons and bells in general have been widely studied by acoustic researchers. Some are interested in the physics of bell sound production [3, 19]. Others have investigated the perception of strike tones [20]. Of particular relevance to this work, [21–25] have made measurements of carillons and investigated the tuning of carillon bells. While some researchers start with physical measurements or models of bells for finite element analysis, we are performing our analysis on single recordings of each bell.

In §2 we discuss the process extracting the modal frequencies, decay rates, and initial complex amplitudes. Next, §3 presents an analysis of the harmonic structure and tuning of the Lurie Carillon, discusses methods for using the analysis data to retune the carillon in the resynthesis process, and proposes novel audio effects that make use of the modal data as a framework. Finally, §4 offers some concluding thoughts.

2. ANALYSIS

There are a number of ways to estimate modal parameters, including Prony’s method [26] and the matrix pencil [27]. In this paper, we use a technique based on successive estimation of frequencies, then decay rates, then complex amplitudes, which uses well-known and fundamental signal processing tools like the FFT, bandpass filtering, and linear least-squares estimation. Similar approaches are used to estimate parameters for modal reverb [12] and other bell studies [21].

¹In this paper, we use the words mode and partial interchangeably.

²An instrument consisting of fewer than 23 bells is called a chime and a grand carillon requires at least 47 bells.

2.1. Modal Approach

We use modal analysis to represent each bell of the Lurie Carillon as a sum of exponentially decaying sinusoids

$$\mathbf{x} = \sum_{m=1}^M a_m e^{j2\pi\omega_m t} e^{-t/\tau_m}, \quad (1)$$

where a_m is the complex amplitude, ω_m the frequency, and τ_m the decay rate for each mode m . An analysis block diagram can be seen in Fig. 1, and the steps for estimating the parameters \mathbf{a} , $\boldsymbol{\omega}$, and $\boldsymbol{\tau}$ are as follows:

1. Perform peak picking in the frequency domain
2. Form band-pass filters around each peak
3. Compute Root-Mean-Squared (RMS) energy envelopes of the band-passed signals
4. Estimate the decay rate on the energy envelopes
5. Estimate the initial complex amplitudes

2.2. Estimating Frequency

The first step in our analysis is to estimate the modal frequencies of each bell. We use a peak picking algorithm in the frequency domain to identify candidate frequencies. Before taking the Fourier Transform for each bell, we high-pass filter the time domain signal half an octave below the hum tone for that bell (see Table 1 for a detailed list of carillon bell modes). The carillon is located outdoors in a high-noise environment and this reduces the likelihood of picking spurious peaks that are simply background noise. Additionally, we use a portion of the time domain bell recording beginning 10ms after the onset so the bell’s noisy transient does not produce a large number of false peaks in the FFT. We use a length 2^{14} Hann window, which produces side-lobes that roll off approximately 18dB per octave.

Our algorithm identifies peaks around points in the frequency domain signal where the slope changes sign from both sides. We then integrate the energy in the FFT bins surrounding the identified peaks and discard peaks that have low power or fall too close to one another. Finally, we pick the N highest candidate peaks that are above some absolute threshold (set by inspection for each bell). For the lowest bells, we estimate around 50 modes and for the highest bells we estimated fewer than ten.

While longer-length FFTs provide better frequency resolution, the fact that we are estimating decaying sinusoids runs counter to this argument. With a long FFT and modes that decay quickly, we will amplify noise that occurs later in the recording. Therefore, the signal to noise ratio is best at the beginning of the recordings. Experimentally, we found 2^{14} samples to be an ideal length across the sixty bells of the carillon, and Fig. 2 shows the results of the peak picking algorithm for several bells.

2.3. Estimating Exponential Decay

We use each frequency found in §2.2 as the center frequency for a fourth-order Butterworth band-pass filter. We find the energy envelope for each partial by averaging the band-pass filtered signals using a 10ms RMS procedure. We then perform a linear fit to the amplitude envelope using least squares to estimate the decay rate of each partial. The region over which the linear fit is performed

Table 1: Partial name and intervalic relationship to the fundamental (prime)

Partial Name	Partial Interval
Hum	Octave (below)
Prime	Fundamental
Tierce	Minor Third
Quint	Perfect Fifth
Nominal	Octave
Deciem	Major Third
Undeciem	Fourth
Duodeciem	Twelfth
Double Octave	Octave
Upper Undeciem	Upper Fourth
Upper Sixth	Major Sixth
Triple Octave	Octave

was found by hand as the bell recordings had large variance of partial-signal-level and noise floor. The result of the slope fitting can be seen in Fig. 3.

2.4. Estimating Complex Amplitude

Once we have estimated the frequency and decay rate of each mode, we estimate the initial amplitude of each partial required to reconstruct the original bell recording. To do this, we form a matrix where each column holds each partial independently as in

$$\mathbf{M} = \begin{bmatrix} 1 & \dots & 1 \\ e^{(j\omega_1 - \tau_1)} & \dots & e^{(j\omega_M - \tau_M)} \\ \vdots & \dots & \vdots \\ e^{(j\omega_1 - \tau_1)T} & \dots & e^{(j\omega_M - \tau_M)T} \end{bmatrix}, \quad (2)$$

where ω_m are the frequencies, τ_m the decay rates, and T is the length of the time vector. We use least squares to find the complex amplitudes

$$\mathbf{a} = (\mathbf{M}^T \mathbf{M})^{-1} \mathbf{M}^T \mathbf{x}, \quad (3)$$

where \mathbf{x} is the original bell recording and \mathbf{a} the vector of complex amplitudes.

2.5. Results

As a result of our analysis, we have estimated frequencies, decay rates, and initial complex amplitudes necessary to model the bells as they were recorded. Fig. 4 plots these parameters for three bells throughout the range of the instrument.

3. RESYNTHESIS

We can resynthesize a “noiseless” copy of the original carillon recordings by directly plugging in the estimated amplitudes, frequencies, and decay rates, into Eq. (1). We can also implement the bell as a filter with the transfer function

$$\frac{X(z)}{U(z)} = \sum_{m=1}^M \frac{2\Re\{a_m\} - 2e^{-1/\tau_m} \Re\{a_m e^{-j\omega_m}\} z^{-1}}{1 - 2e^{-1/\tau_m} \cos(\omega_m) z^{-1} + e^{-2/\tau_m} z^{-2}}, \quad (4)$$

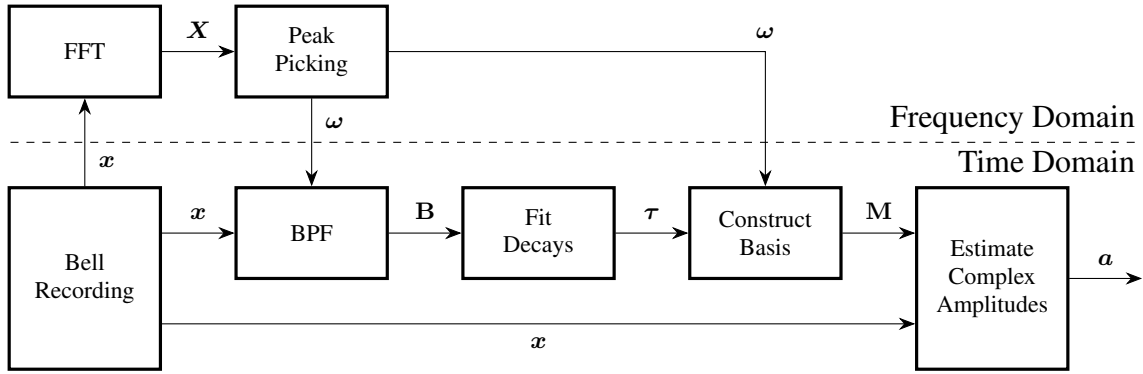


Figure 1: Block diagram detailing the analysis process for finding the modal frequencies (ω), decay rates (τ), and complex amplitudes (a).

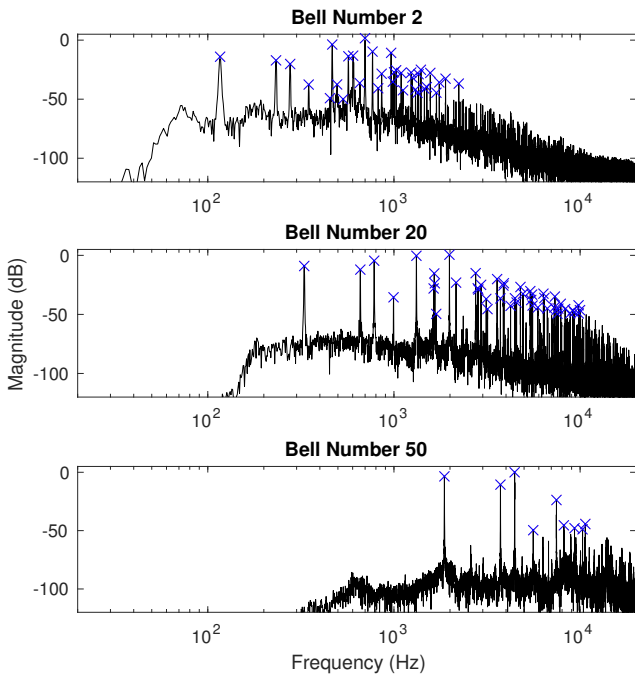


Figure 2: Frequency spectrum of three Bells from the Lurie Carillon showing the results of the peak picking algorithm (in blue).

where U is the input. This is the same as a parallel sum of M standard second order (biquad) transfer functions

$$\frac{X(z)}{U(z)} = \sum_{m=1}^M \frac{\beta_{0,m} + \beta_{1,m}z^{-1} + \beta_{2,m}z^{-2}}{1 + \alpha_{1,m}z^{-1} + \alpha_{2,m}z^{-2}} \quad (5)$$

with coefficients

$$\begin{aligned} \beta_{0,m} &= 2\Re\{a_m\} \\ \beta_{1,m} &= -2e^{-1/\tau_m} \Re\{a_m e^{-j\omega_m}\} & \alpha_{1,m} &= -2e^{-1/\tau_m} \cos(\omega_m) \\ \beta_{2,m} &= 0 & \alpha_{2,m} &= e^{-2/\tau_m} \end{aligned}$$

If U is an impulse, we reconstruct the sound of the bell, and Eq. (4) is equivalent to Eq. (1). Both synthesis models take the form of summing modes as seen in Fig. 5.

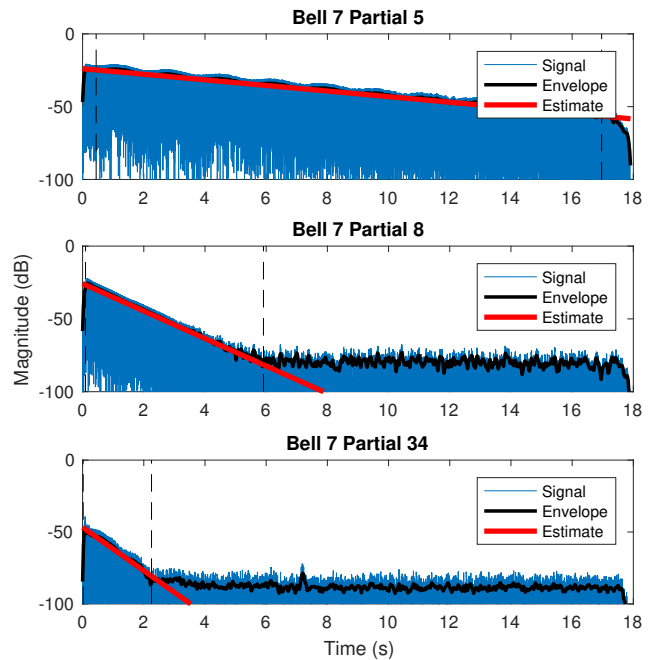


Figure 3: Estimated decay rates for three partials of bell seven. Each plot shows the recorded bell signal band-pass filtered around the partial frequency (blue), the RMS smoothed energy envelope (black), and the estimated decay rate (red). Dotted lines show the region over which the linear fit was performed.

We find that thirty modes for a low-pitched bell and ten for a high-pitched bell do a reasonable job reproducing the sound of the recordings. Using the modal data extracted from the recordings of the Lurie Bells, we find the correlation coefficient between the recorded bells and synthesized bells was on average 0.837 with a standard deviation of 0.117. Fig. 6 shows the result of this resynthesis for one of the bells of the Lurie Carillon. One major advantage to the resynthesis is that the real bells ring for a significantly long time. The recordings faded out once the bell is indistinguishable from the noise floor. The resynthesis does not have this limitation and is therefore more impervious to amplitude modifications.

As a result of our analysis, we have a parameterized model rather than simply a means for synthesizing the sound of the orig-

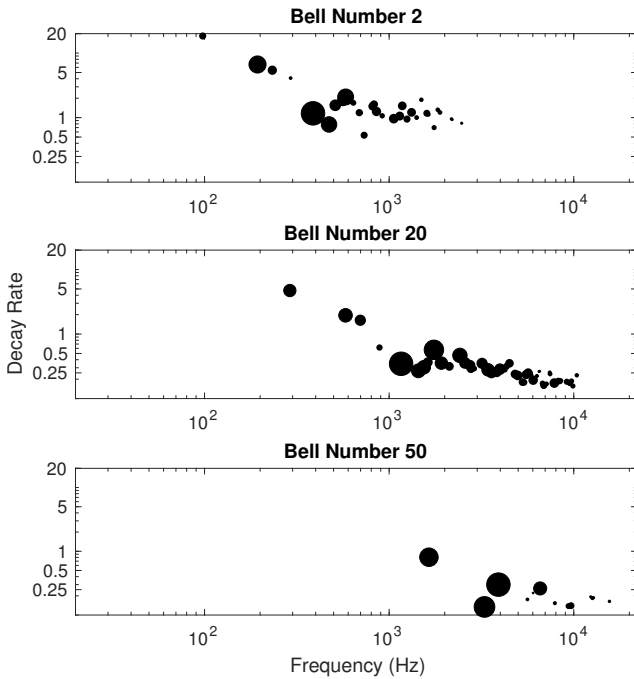


Figure 4: Frequency vs decay rate for three analyzed bells with marker diameter showing each partial’s amplitude.

inal recordings. In addition to providing a way to study the harmonic structure and tuning of the Lurie Carillon, we can manipulate the model to synthesize bell sounds with different characteristics or bell-based audio effects. The analysis/resynthesis diagram can be seen in Fig. 7. The modifications we make in the resynthesis primarily involve scaling and adding offsets to the estimated frequencies, amplitudes, and decay rates, as well as the use of external control data. Some bell related processing is described below. Audio examples and code to produce them can be found at <https://ccrma.stanford.edu/~kermit/website/bells.html>.

3.1. Computer Aided Electronic Bell Foundry

Carillons bells are designed to function together as a cohesive instrument, however the range of the instrument, the process of casting bronze bells, and the way the instrument wears when played make this challenging. The Lurie Carillon has sixty bells that span five octaves. The largest bells weigh up to 5000kg while the smallest weigh only 5kg. Naturally this means the bells across the instrument are cast with different shapes and it takes skill and patience to tune the instrument as a whole. Furthermore, the bells are often exposed to extreme weather conditions as they are located in semi-open spaces atop towers. Over time the bells do not necessarily wear in the same way due to how often each is struck and its independent location within the belfry. Last, bell foundry technology and playing techniques have developed over time [28]. Bells cast by each manufacture have subtle differences, and while most modern instruments are tuned to equal temperament, this is not the case for older instruments.

For these reasons, we propose a collection of manipulations using the modal data from the Lurie Carillon to modify the sound

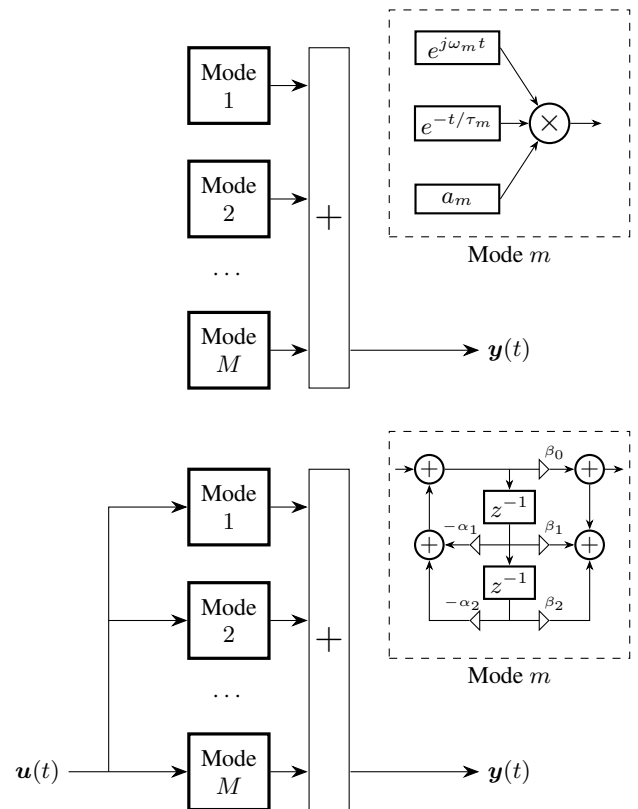


Figure 5: Direct resynthesis (top) and biquad (bottom) models.

in ways that could be relevant to carillonists and musicologists. Fig. 8 shows all the bells of the Lurie Carillon, pitch-shifted to share a common fundamental. From inspection, one can see that the lowest two partials are in tune across the whole instrument but the quint (fourth partial) starts going sharp to the point that it is almost half an octave sharp in the highest bells. Additionally, the highest bells have partials that are significantly flatter than the other bells. Even so, the bells of the instrument sound well matched lending precedence to the fact that one cannot use a single bell and pitch shift it up and down to synthesize bells at all pitch-heights.

3.1.1. Fixing Irregularities Due to Wear

Tuning carillon bells is an arduous process. The procedure often involves turning the bells on a lathe to evenly remove material at specific points on the bell to sharpen specific partials. Modern technology has improved bell tuners’ ability to make carillon bells sound more homogeneous, however, material can only be removed. This makes it impossible to flatten the pitch of a bell. Furthermore, even though the bells are designed to sound like the rest of the bells of a specific instrument, the nature of bell casting makes this idealism impossible. With our analysis of the full set of bells from the Lurie Carillon, we can average the modal parameters of several consecutive bells in order to smooth out any irregularities caused by a poorly tuned bell.

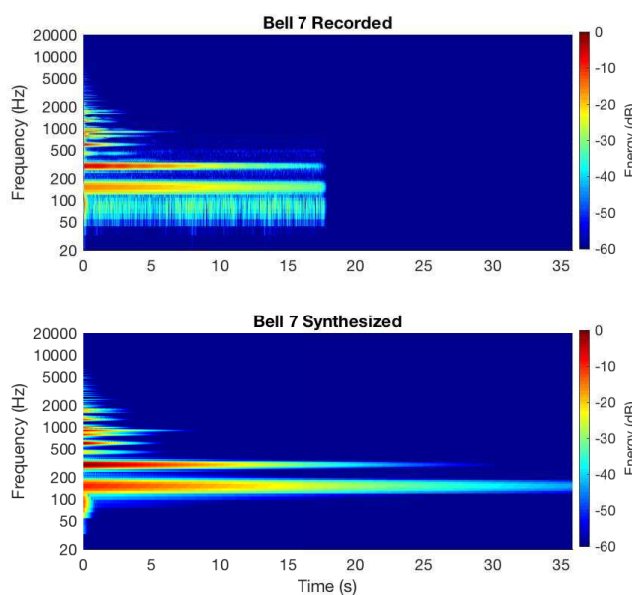


Figure 6: Spectrograms showing original recording and resynthesis of bell number seven.

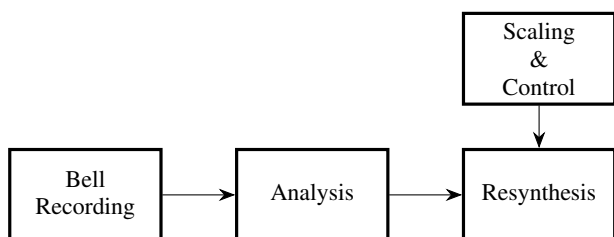


Figure 7: Analysis/Resynthesis block diagram.

3.1.2. Retuning the Carillon

In addition to making the carillon bells sound more consistent with one another, we can also retune the entire carillon. While modern instruments are tuned in equal temperament, there is an enormous repertoire that was written for mean-tone and other tuning schemes. Some music was written to be played in mean-tone temperament and does not sound as the composer intended when played in equal temperament. With the modal data, we can make two types of adjustments. First, we can resynthesize a set of carillon bell samples with the same characteristics as the Lurie Carillon but shift the frequencies of each bell’s partials such that the fundamentals are tuned to a scheme other than the current equally tempered bells. Second, we can modify the tuning within each bell, correcting irregularities and ill-tuned partials.

3.1.3. Extending the Range of the Carillon

The Lurie Carillon consists of sixty bells, however there are larger instruments. For example, the Laura Spelman Rockefeller Memorial Carillon at the Riverside Church in New York City has a carillon with 74 bells [29]. Carillons this large are rare as they are expensive to construct and the towers that hold them must be able

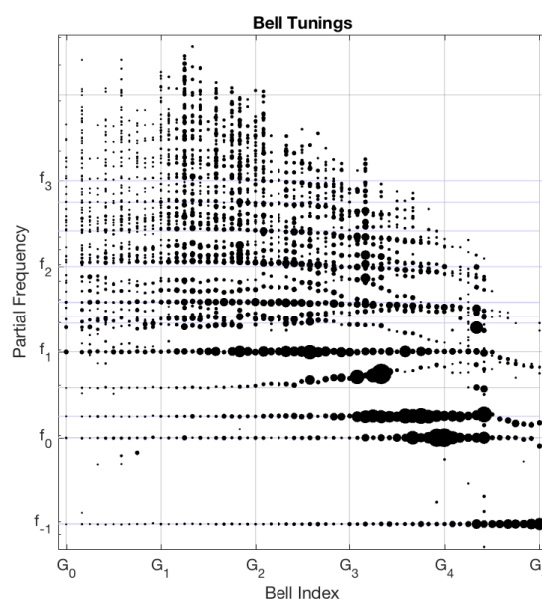


Figure 8: Estimated parameters of all the bells of the Lurie Carillon with common fundamental. Marker diameter corresponds to bell-power integrated over the first second. The measured partial frequencies are overlaid on the theoretical 12-tone equal temperament tunings (blue lines).

to support the immense weight of such a heavy instrument. Moreover, the Lurie Carillon, like many instruments, is missing the second largest bell for the above mentioned reasons and the fact that it is rarely needed to perform the standard carillon literature. By extrapolating from the measured modal data, we can virtually extend the range of the instrument to incorporate missing bells and ones higher or lower in pitch than the physical bells. Additionally, we can interpolate between bells to produce notes other than the 12-tone equally tempered pitches. This allows us to synthesize, for example, a set of quarter-tone bells that would be useful to contemporary composers, but would be impractical to cast otherwise.

3.1.4. Adapting the Decay Rate

The tradition of writing and performing carillon music has naturally progressed alongside the technical developments of bell casting and carillon construction. In many ways, the style of carillon music has worked around physical limitations of the acoustic instrument. One fundamental property of a carillon is that the higher pitched bells ring for a shorter amount of time than the lower ones. A Dutch technique was developed for playing slower tempo pieces that utilizes playing tremolo and trill gestures in the high bells to mimic a sustain [30]. Some carillonists find this technique displeasing to the ear. To overcome the fact that the high bells decay much faster than the lower bells, we can modify the decay rates of the higher bells so they last a similar length as the lower bells. This would allow one to play legato pieces and circumvent the physical limitation inherent in the physical instrument. We can also render the low bells with a quicker decay rate so that they can be played faster without sounding muddy to their natural, long decay rate.

3.1.5. Major Third Bells

Until recently (1980s), the spectrum produced by most bells has prominently featured a minor third.³ This pronounced harmonic structure caused bell-to-bell harmonies to often have a "rough" sound due to the clashing of the tunings. Due primarily to the immense construction cost, there are very few major-third carillons in existence. It is rather humorous, but carillonists are used to the sound of old bells and mostly prefer the traditional sound while modern composers say they prefer major-third bells for major-key pieces and minor-third bells for minor-key pieces [30]. It is easy for us to resynthesize the Lurie Carillon samples with the partial related to the minor third modulated up a half-step.

3.2. Electroacoustic Modifications

In addition to manipulating the modal parameters and synthesizing exponentially decaying bell sounds, we can produce a host of time-varying bell effects that could be useful to electroacoustic composers.

3.2.1. Adapt the Decay Rates

A limitation that contemporary composers writing for carillon often encounter is the speed at which the low bells can be re-struck. Physically, there is a minimum time necessary to move the clapper into the bell. Ignoring that, the long ring time of the lowest bells makes the sound of re-striking the bell blend together. The articulations become indistinguishable from the previous strike's sustain. If one simply scales the decay rates of all the partials of a bell, the resulting sound appears to be low-pass filtered as the high frequencies would decay quickly. To improve the result, we propose scaling the decay rates nonlinearly to only shorten the ring time of the lowest frequencies. This affords a composer the ability to re-trigger the low bells quickly without the transients becoming blurred out and without the whole bell sounding filtered.

3.2.2. Add Arbitrary Amplitude Envelopes

Typically, bells are characterized by an exponentially decaying energy envelope. In resynthesis, we can use the estimated frequencies and replace the amplitude envelopes with arbitrary functions (e.g., ADSRs). In effect, this allows us to allude to the harmony of the carillon bells without producing sounds that decay exponentially like the bells.

3.2.3. Spectral Morphing

In his magnum opus, *Mortuos Plango, Vivos Voco*, Jonathan Harvey uses the spectrum of the tenor bell from the Winchester Cathedral as the basis for the harmony and structure of his piece [31]. Using the modal data, we can morph between various bells by connecting the partials together with glissandi. In Harvey's piece, the composer resynthesizes the individual partials of the bell using a pitch-shifted sample of a boy's voice. We too can use the extracted frequencies for each partial to control other musical parameters, such as the rate at which a buffer is read, the pitch of a synthesizer, or the center frequency for a filter.

³Unlike the harmonic spectrum produced by most musical instruments.

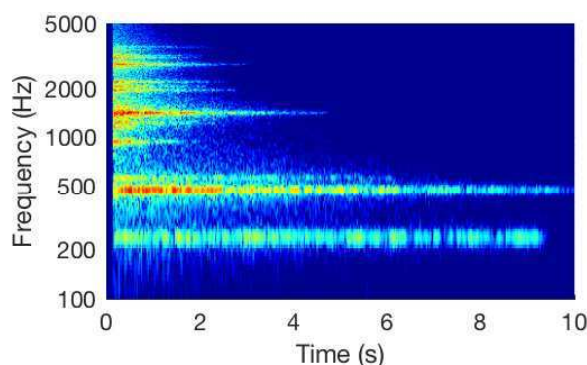


Figure 9: Spectrogram of exponentially-enveloped Gaussian noise passed through band-pass filters centered around the estimated modal frequencies.

3.2.4. Bells as Reverb

Our default synthesis model resynthesizes each partial as an exponentially decaying sinusoid. Instead, we can pass Gaussian noise through band-pass filters, each centered around the bell-partial frequencies. We can additionally control the Q factor of the filters in order to control the bandwidth of the noise at each partial. Alone, this resynthesis creates an airy sound effect that has bell-like characteristics (see Fig. 9), but we can also use it as a reverberation effect applied to other sounds.

3.2.5. Add Modulation and Doublets

Natural sounding bells have beating due to minor imperfections. Across the measured bells, there is a high variance for how much beating there is (both how different the frequencies are and how strongly the two modes differ in amplitude and phase). In the resynthesis, we can specifically control how much beating occurs on a partial-by-partial basis. We can achieve this in two ways. First, for each measured partial we can synthesize two partials that have small differences in the modal parameters. Second, we can apply a low frequency oscillator (LFO) to each partial to apply amplitude or frequency modulation. Both approaches allow us to synthesize natural sounding bell sounds. By applying severe detuning or modulation, we can create additional effects (e.g., vibrato, tremolo, FM distortion, etc). Moreover, we can change this effects dynamically. For example, we can have the sound of a bell that has no modulation, one second into the decay apply vibrato, and two seconds into the decay increase the modulation depth and speed to create distortion.

3.2.6. Spatialization and Time Modifications

Since we have control of each partial, we can modulate their spatial positioning and onset timing independently. Thus we can deconstruct the bell into an arpeggio, randomize the entrance time for each partial, or spread the bell across the stereo field.

3.3. Summary

Fig. 10 shows a full sound example using several of the techniques described above. At the beginning, the bell is synthesized three times with each partial entering at a different time. Over the first

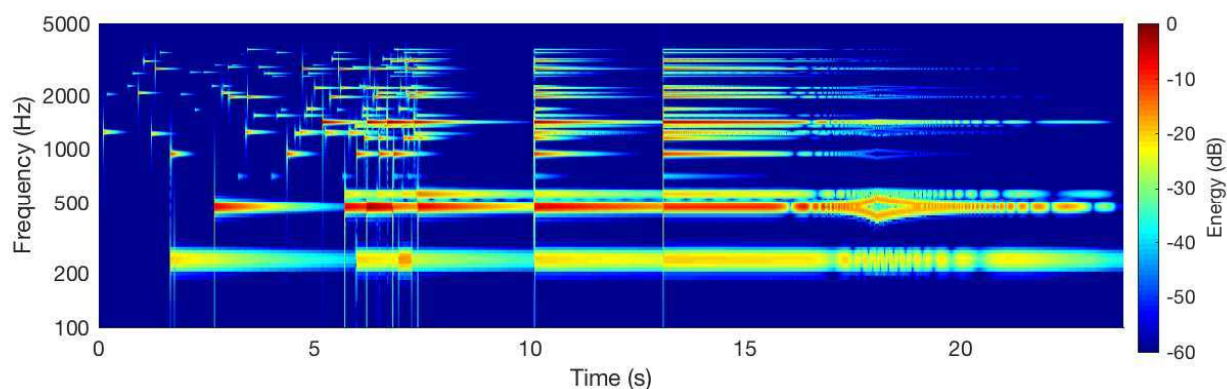


Figure 10: Spectrogram of a through composed example demonstrating scaling and modulation effects.

eight seconds, the timing between each partial’s entrance decreases and the decay rates for each partial is slowly lengthened. At ten seconds, the unmodified bell is sounded. The bell is struck once more around second twelve with a long decay and a modulation applied on each partial. The modulation depth is increased until a frequency modulation effect is apparent before slowing back to a vibrato.

Overall, this is not a comprehensive list of modal resynthesis techniques. In fact, this exploration just shows that the range of sonic possibilities is enormous, even when constrained by a simple model consisting of frequency, decay rate, and initial amplitude.

4. CONCLUSIONS

In this paper we demonstrate how a modal model for sound synthesis can be manipulated to achieve a wide range of musical effects. Using the carillon as a case study, we provide an analysis of the University of Michigan’s Lurie Carillon. By modeling each bell as a sum of decaying sinusoids at each modal frequency, we provide a versatile model that lends itself well to a host of audio manipulations consisting primarily of scaling and shifting the parameters. As a tool for composers, we hope this analysis of the Lurie Carillon will help encourage new electroacoustic music for carillon. The modal data generated from analyzing the samples from [15] is freely available under a creative commons license and can be downloaded at <https://ccrma.stanford.edu/~kermit/website/bells.html>.

The method used in this paper has a limited ability to resolve very closely spaced partials, known as “doublets” in the bell context. Future work will attempt to estimate doublet parameters using nonlinear optimization. An alternate approach to modeling doublet behavior is shown in [32, 33], where groups of closely-spaced modes are approximated using ARMA modeling.

5. ACKNOWLEDGMENTS

The authors thank Jonathan Abel for his helpful comments.

6. REFERENCES

[1] Stefan Bilbao, *Numerical Sound Synthesis*, Wiley, 2009.

[2] Vesa Välimäki, Julian D. Parker, Lauri Savioja, Julius O. Smith, and Jonathan S. Abel, “Fifty years of artificial reverberation,” *IEEE Transactions on Audio, Speech, and Language Processing*, vol. 20, no. 5, pp. 1421–48, July 2012.

[3] Tomas Rossing and Robert Perrin, “Vibrations of bells,” *Applied Acoustics*, vol. 20, pp. 41–70, 1987.

[4] Lutz Trautmann and Rudolf Rabenstein, *Digital Sound Synthesis by Physical Modeling Using the Functional Transform Method*, Springer, New York, 1st edition, 2003.

[5] Gerhard Eckel, Francisco Iovino, and René Caussé, “Sound synthesis by physical modelling with Modalys,” in *Proceedings of the International Symposium on Musical Acoustics*, Dourdan, France, 1995.

[6] Joseph Derek Morrison and Jean-Marie Adrien, “MOSAIC: A framework for modal synthesis,” *Computer Music Journal*, vol. 17, no. 1, pp. 45–56, Spring 1993.

[7] Maarten van Walstijn and Jamie Bridges, “Simulation of distributed contact in string instruments: A modal expansion approach,” in *Proceedings of the 24th European Signal Processing Conference (EUSIPCO)*, Budapest, Hungary, August 29 – September 2 2016, pp. 1023–7.

[8] Maarten van Walstijn, Jamie Bridges, and Sandor Mehes, “A real-time synthesis oriented tanpura model,” in *Proceedings of the 19th International Conference on Digital Audio Effects (DAFx-16)*, Brno, Czech Republic, September 5–9 2016, pp. 175–82.

[9] Clara Issanchou, Stefan Bilbao, Cyril Touze, and Olivier Doare, “A modal approach to the numerical simulation of a string vibrating against an obstacle: Applications to sound synthesis,” in *Proceedings of the 19th International Conference on Digital Audio Effects (DAFx-16)*, Brno, Czech Republic, September 5–9 2016.

[10] Esteban Maestre, Gary P. Scavonne, and Julius O. Smith, “Design of recursive digital filters in parallel form by linearly constrained pole optimization,” *IEEE Signal Processing Letters*, vol. 23, no. 11, pp. 1547–50, 2016.

[11] Jonathan S. Abel, Sean Coffin, and Kyle S Spratt, “A modal architecture for artificial reverberation,” *The Journal of the Acoustical Society of America*, vol. 134, no. 5, pp. 4220, 2013.

- [12] Jonathan S. Abel, Sean Coffin, and Kyle Spratt, “A modal architecture for artificial reverberation with application to room acoustics modeling,” in *Proceedings of the 137th Convention of the Audio Engineering Society*, Los Angeles, CA, October 9–12 2014.
- [13] Jonathan S. Abel and Kurt James Werner, “Distortion and pitch processing using a modal reverberator architecture,” in *Proceedings of the 18th International Conference on Digital Audio Effects (DAFx-15)*, Trondheim, Norway, November 30 – December 3 2015.
- [14] Kurt James Werner and Jonathan S. Abel, “Modal processor effects inspired by Hammond tonewheel organs,” *Applied Sciences*, vol. 6, no. 7, pp. 1421–48, 2016, Article #185.
- [15] Isaac Levine, Ashton Baker, Rowena Ng, Rachael Park, Anjana Rajagopal, Tiffany Ng, and John Granzow, “Download Lurie carillon samples,” <https://gobluebells.wordpress.com/2016/09/21/lurie-carillon-samples/>.
- [16] “World carillon federation,” <http://www.carillon.org/>.
- [17] “Lurie carillon,” https://www.music.umich.edu/about/facilities/north_campus/lurie/lurie.htm.
- [18] “Ann & Robert H. Lurie Carillon,” <http://www.towerbells.org/data/MIANNAU2.HTM>.
- [19] Robert Perrin and T. Charnley, “A comparative study of the normal modes of various modern bells,” *Journal of Sound and Vibration*, vol. 117, no. 3, pp. 411–420, 1987.
- [20] William Hibbert, *The Quantification of Strike Pitch and Pitch Shifts in Church Bells*, Ph.D. thesis, The Open University, 2008.
- [21] Xavier Boutillon and Bertrand David, “Assessing tuning and damping of historical carillon bells and their changes through restoration,” *Applied Acoustics*, vol. 64, pp. 901–10, 2001.
- [22] Vincent Debut, Miguel Carvalho, and Jose Antunes, “An objective approach for assessing the tuning properties of historical carillons,” in *Proceedings of the Stockholm Music Acoustics Conference*, Stockholm, Sweden, July 2013.
- [23] Wiegman van Heuven, *Acoustical measurements on church-bells and carillons*, Ph.D. thesis, De Gebroeders van Cleef, The Hague, Netherlands, 1949.
- [24] Andre Lehr, “The system of the Hemony-carillons tuning,” *Acustica*, vol. 3, pp. 101–104, 1951.
- [25] Albrecht Schneider and Marc Leman, *Studies in Musical Acoustics and Psychoacoustics*, chapter Sound, Pitches and Tuning of a Historic Carillon, pp. 247–98, Springer, 2016.
- [26] T. W. Parks and C. S. Burrus, *Digital Filter Design*, Wiley-Interscience, New York, NY, USA, 1987.
- [27] Y. Hua and T. K. Sarkar, “Matrix pencil method for estimating parameters of exponentially damped/undamped sinusoids in noise,” *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 38, no. 5, pp. 814–24, May 1990.
- [28] Andre Lehr, “Contemporary Dutch bell-founding art,” *Netherlands Acoustical Society*, , no. 7, pp. 20–49, 1965.
- [29] “The Riverside Church carillon,” www.trcnyc.org/events/carillon?/.
- [30] Brian Swager, *A History of the Carillon: Its origins, Development, and Evolution as a Musical Instrument*, Ph.D. thesis, Indiana University, 1993.
- [31] Jonathan Harvey, “Mortuos Plango, Vivos Voco: A realization at IRCAM,” *Computer Music Journal*, vol. 5, no. 47, 1981.
- [32] Matti Karjalainen, Paulo A. A. Esquef, Poju Antsallo, Aki Mäkivirta, and Vesa Välimäki, “Frequency-zooming ARMA modeling of resonant and reverberant systems,” *Journal of the Audio Engineering Society*, vol. 50, no. 12, pp. 1012–29, December 2002.
- [33] Matti Karjalainen, Vesa Välimäki, and Paulo A. A. Esquef, “Efficient modeling and synthesis of bell-like sounds,” in *Proceedings of the 5th International Conference on Digital Audio Effects (DAFx-02)*, Hamburg, Germany, September 25–28 2002, pp. 181–6.